

# A Guide to Fixed Income Portfolio Management using Risk Duration, Reward Duration, and Duration Ratio as alternatives to Effective Duration and Convexity

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**Abstract.** -Macaulay's duration (a.k.a. "duration") and modified duration are not effective risk management measurements on callable or pre-payable bonds. They are fine on non-callable and non-pre-payable bonds.

-“Effective duration” is useful on callable and pre-payable bonds, HOWEVER, it is useless by itself. Convexity MUST be combined with effective duration in order to be of value in fixed income risk management.

-Effective duration and convexity combined, though technically accurate, are very difficult to interpret and apply in fixed income interest rate risk management.

-Alternatives to effective duration and convexity that offer the same valuable risk management information, but in a much simpler, intuitive and usable form are Risk Duration, Reward Duration, and Duration Ratio.

-Risk Duration and Duration Ratio are the two most powerful statistics available to today's fixed income portfolio and risk managers.

**Key words.** Risk Duration, Reward Duration, Duration Ratio, Macaulay duration, modified duration, duration, convexity, Allen's convexity, quality ratio, fixed income portfolio management, bond management, risk management.

***Historical Recap of the Term “Duration”: Measurement of time or price sensitivity?***

In 1938, Frederick Macaulay introduced “duration” in the bond industry in an effort to estimate price sensitivity due to interest rate changes on non-callable bonds. This measurement was later called “Macaulay's duration.” “Duration” is defined by Webster's dictionary as:

1: continuance in *time*

2: the *time* during which something exists or lasts

Since Macaulay's formula technically measures time (weighted average maturity of a bond's cash flows), the term “duration” was appropriate. However, price sensitivity is the primary benefit of Macaulay's formula, not time, thus “duration” turned out to be a misnomer because price sensitivity is not a measurement of time. Macaulay's duration was later adjusted to better estimate a non-callable bond's price sensitivity to changing interest rates with the advent of “modified duration.” **Modified duration is NOT a measurement of time**, but a measurement of price sensitivity due to changing interest rates.

The abbreviated term “duration” is commonly used as a substitution for: Macaulay's duration (properly), modified duration (improperly), and effective duration (improperly). Despite the mismatch in terminology, the term “duration” has attached itself to two measurements that do not measure time at all: modified duration and effective duration.

### ***Modified duration and convexity***

For non-callable and non-pre-payable bonds (U.S. Treasuries, for example), interest rate risk and reward can be closely approximated by applying the bond's *modified duration*. Interest rate risk is how much market value the bond stands to lose given a specific higher (usually +1%) interest rate scenario. The reward is how much market value the bond stands to gain given a specific lower (usually -1%) interest rate scenario. These risk and reward numbers are measured in percentage terms (not in years). In general, *modified duration* / 100 is the percentage market value that the bond stands to gain (in lower interest rate scenario) and *-modified duration* / 100 is the percentage market value that the bond stands to lose (in higher interest rate scenario).

These risk and reward percentages can be measured in a more exact fashion by adding another technical feature called *convexity*. *Convexity* on non-callable (including non-pre-payable) bonds is “positive,” meaning the value of the bond rises a bit more than *modified duration* / 100 in a lower interest rate scenario. It also means the value of the bond falls a bit less than *-modified duration* / 100 in a higher interest rate scenario. *Modified duration* and *convexity* are somewhat complex mathematical formulas, but are very common and are widely available. Many financial calculators and spreadsheet programs (i.e., Excel) perform calculations on *modified duration* and *convexity*. For a given set of inputs (maturity term, coupon, yield) there is only one correct answer for modified duration and convexity. There is no subjectivity.

Historically, these calculations were all fixed income portfolio managers and bond investors needed to measure interest rate risk. However, with the advent of mortgage-backed bonds that are pre-payable at the debtor's option and the preponderance of call options on many (if not most) corporate, municipal and government agency debentures, *modified duration* proved to be ineffective in accurately estimating a callable bond's price movements. On a callable bond, for instance, there are effectively two possible maturity dates: stated maturity date (typically applied with increasing interest rates) and call date (typically applied with decreasing interest rates). It is possible to calculate two separate modified duration calculations, one for the maturity date and one for the call date. Many managers do this, but there is not much value in using modified duration on callable bonds. This is because modified duration will not take into account real world factors unique to callable bond pricing such as "spread tightening."

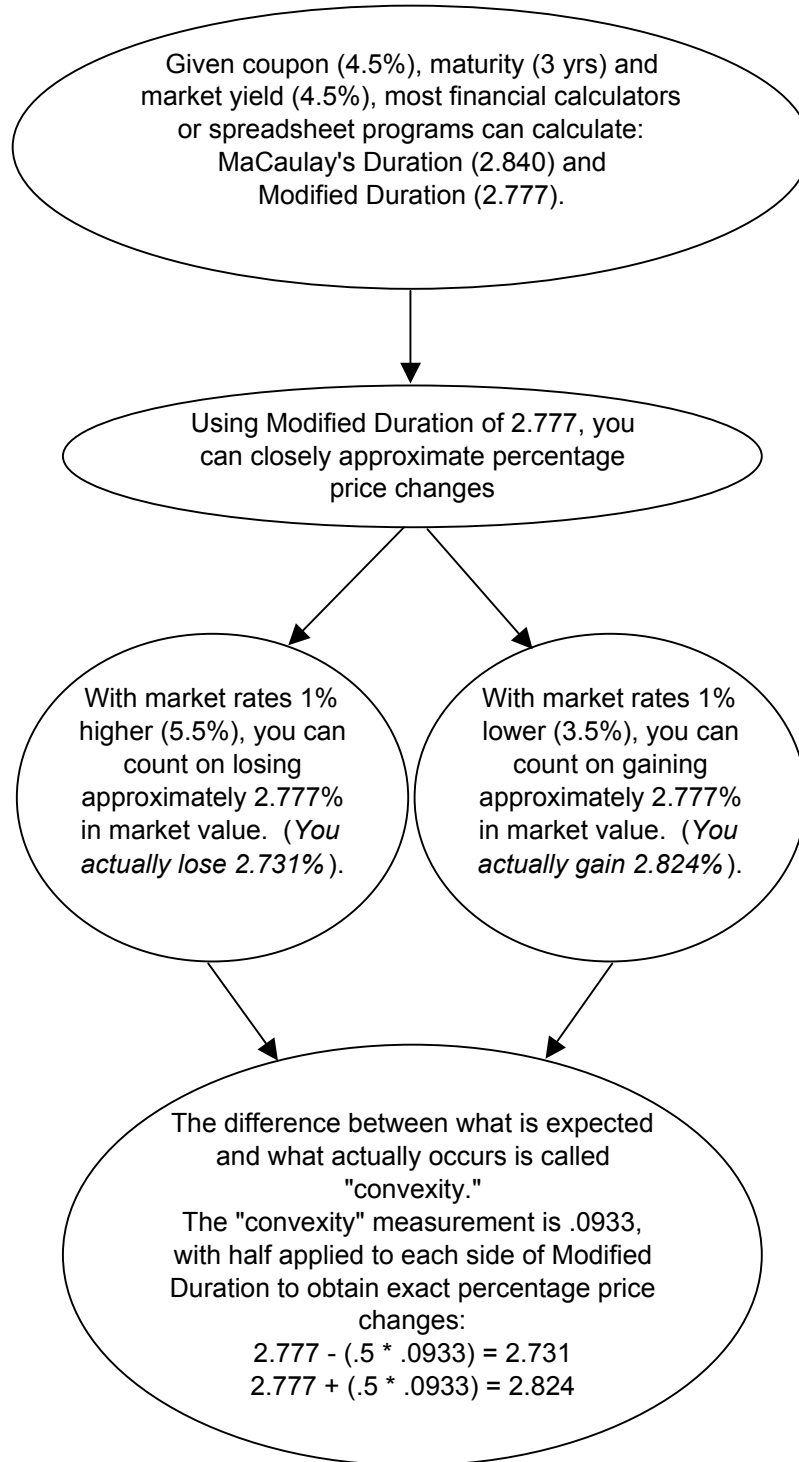
When interest rates in general increase 1%, for example, the market yield on a callable bond will not typically increase the full 1% like a non-callable bond would. The market interest rate on the actual bond may only increase .85% (85 basis points) versus a full 1% (100 basis points) for general interest rates. This would be a spread tightening of 15 basis points. Spread tightening and volatility, which is a key component in spread tightening, are both subjective and vary from pricing source to pricing source. Prepayment speeds on mortgage backed

securities are another key input into market pricing that varies from source to source due to its subjectivity.

An appropriate analogy on non-callable and non-pre-payable bonds is *modified duration* “kills two birds with one stone”. Picture the two birds representing the absolute value percentage price changes in up and down interest rate scenarios. On non-callable and non-pre-payable bonds, the birds fly close to each other, so given the coordinates (modified duration) of the birds, one can toss a stone in the proper direction and kill both birds. This is very convenient.

<<The next page contains a flow chart illustrating content from above info>>

These measurements are what you use for non-callable, non-prepayable bonds. Treasury notes and other "bullets" will be relevant in this sequence, while callable or prepayable bonds will not.



## ***Effective Duration and Effective Convexity***

*Effective duration* and *effective convexity* (a.k.a. “convexity”) have been, to date, the best interest rate risk and reward measurements for callable and pre-payable bonds. Combined in a very technical and exact (and to many, very confusing and less than intuitive) manner, these two measurements show how much a bond stands to gain in a specific lower interest rate scenario (e.g. -1%) (reward) and how much the same bond stands to lose in a specific higher interest rate scenario (e.g. +1%) (risk).

It is crucial to know that given coupon, maturity, and yield, there is no standard formula for *effective duration* and *effective convexity* like there is for *modified duration* and *convexity*. One cannot punch these numbers into to a financial calculator and retrieve *effective duration* and *effective convexity*. One must first calculate how much market value a callable or pre-payable bond stands to lose and gain in equal and opposite interest rate shifts (e.g. +/- 1%). Only then can *effective duration* and *effective convexity* be figured.

Consider a callable bond with 3-year maturity and 1-year call option that is currently worth 100. Rigorous analyses such as option adjusted calculations using Monte Carlo simulations or other proprietary simulation models used to estimate the bond’s market value at various interest rate scenarios must be used. A financial calculator will not do it. There are too many subjective inputs required for a standard formula: implied volatility and prepayment speeds on mortgage or

other asset backed securities are two big factors to be considered that are not standard, observable inputs. They are subjective. If a bond manager, or another source, determines that the market price would be 98 if interest rates rose 1%, then the percentage price change would be -2%. If it is determined that the market price in an equal and opposite move in interest rates (down 1%) results in a market price of 101, the percentage price change would be +1%.

Armed with potential percentage price movements:

*Effective duration* =

(-(up interest rate scenario's percentage price change from current interest rates) + down interest rate scenario's percentage price change from current interest rates) / 2 \* 100.

*Effective convexity* =

(down interest rate scenario's percentage price change from current interest rates + up interest rate scenario's percentage price change from current interest rates) \* 100.

In the example above:

*effective duration* =  $(-1 * -2\% + 1\%) / 2 * 100 = \underline{1.5}$

*effective convexity* =  $(1\% + -2\%) * 100 = \underline{-1}$ .

*Effective duration* is the average percentage market value change on the bond given equal and opposite interest rate shifts (e.g. +/- 1%). From a 1.5 *effective duration* measurement and current market price of 100, the price in an up 1% interest rate scenario is initially estimated to be 98.5 (-1.5%). The price in a down 1% interest rate scenario is initially estimated to be 101.5 (+1.5%). Unfortunately, on callable and pre-payable bonds, *effective duration* alone is not a good enough measurement of interest rate risk and reward. *Effective convexity* must be included to provide information that is usable to a fixed income portfolio manager.

*Effective convexity* (a.k.a. “convexity”) is the total error of this average value move (*effective duration*). How much does *effective duration* need to be adjusted? From a -1 *effective convexity* measurement, take half of this and apply it to each estimated price movement. Instead of market value increasing by 1.5% (as estimated by *effective duration*), it increases: 1.5% less half of the total error (*effective convexity*), or +1%. Instead of market value decreasing by 1.5% (as estimated by *effective duration*), it decreases 1.5% plus half of the total error (*effective convexity*), or -2%. If *effective convexity* is negative, which it is on virtually all callable or pre-payable bonds, the extra percentage price change (*effective convexity*) works against both scenarios’ price change estimations suggested by *effective duration*.

Once *effective duration* and *effective convexity* are derived from assumed price change movements, what does *effective duration* and *effective convexity* tell us? It tells us what percentage market price changes we should expect. The formulas are:

Percentage price change for (assumed) 1% shift up in interest rates =  $((.5 * \textit{effective convexity}) - \textit{effective duration}) / 100 = ((.5 * -1) - 1.5) / 100 = -2\%$ .

Percentage price change for (assumed) 1% shift down in interest rates =  $(\textit{effective duration} + (.5 * \textit{effective convexity})) / 100 = (1.5 + (.5 * -1)) / 100 = +1\%$ .

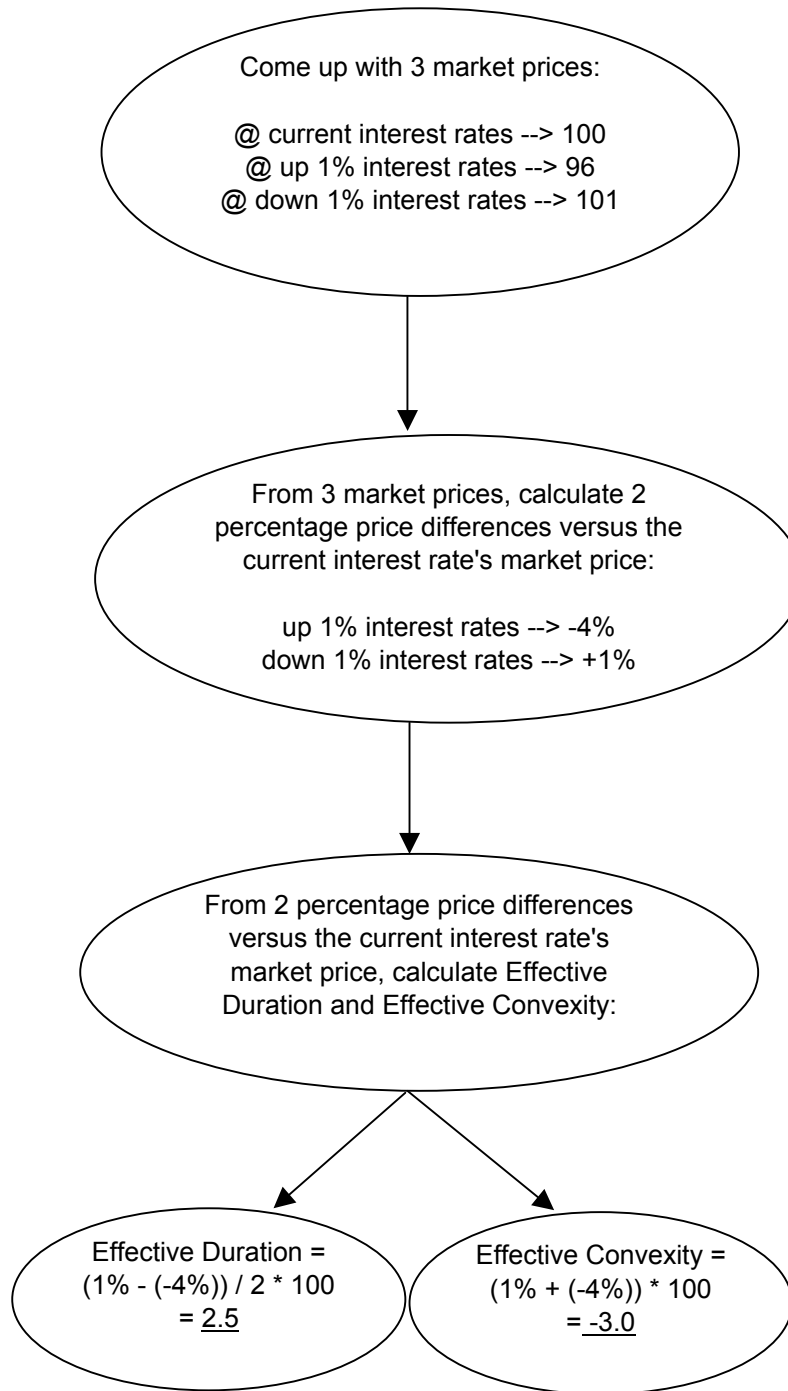
Feel like we just went full circle? We did! *Effective duration* and *effective convexity* require percentage market value changes and then *effective duration* and *effective convexity* are used to gauge percentage market value changes. It is circular, though not useless. *Effective duration* and *effective convexity*, as a pair, are alternative ways of stating up and down interest rate scenario's percentage market price changes.

Adding to the previous “two birds with one stone” analogy, on callable and pre-payable bonds, the “birds” do not typically fly very close to each other like they do on non-callable and non-pre-payable bonds. Imagine one bird flying at 11:00 and the other at 1:00 (30 degrees left and right of center, or directly overhead).

The average of these two coordinates is high noon, or 12:00 (directly overhead). Using 12:00 noon (*effective duration*) as our lone coordinate, a tossed stone will miss both birds. Two stones AND adjustments to 12:00 noon (*effective duration*) are needed in order to kill both birds. The aim needs to be adjusted a total of 60 degrees (*effective convexity*) to the left and right of 12:00 noon (*effective duration*). One aim needs to be degrees left of 12:00, or 11:00, and the other aim needs to be 30 degrees right of 12:00, or 1:00. Now two birds can be killed with two stones. This is the best one can do now that pairs of birds tend to NOT fly together anymore (callable and pre-payable bonds).

<<The next 2 pages contain flow charts illustrating content from above info>>

Given 3 interest rate scenario's market prices, this is how you come up with Effective Duration and Effective Convexity:



Given Effective Duration and Effective Convexity, you can tell how much a bond's value stands to increase or decrease:

Effective Duration = 2.5

Effective Convexity = -3

Given Effective Duration and Effective Convexity, calculate 2 percentage price differences from current market price:

up 1% market price  
difference from current market  
price (as %) =  
 $((1/2 * -3) - 2.5) / 100$   
= -4%

down 1% market price  
difference from current market  
price (as %) =  
 $(2.5 + (1/2 * -3)) / 100$   
= +1%

### **Common misuses of “duration”**

“Duration” (Macaulay’s duration, *modified duration*, *effective duration*) has been a way of generally referring to the interest rate risk (and reward) of a bond, i.e., market value gain or loss given a specific interest rate shift. Terms such as “average life” on amortizing bonds (e.g. mortgage backed securities—“MBS”), “workout,” “expected maturity,” and “expected call date” refer to the expected principal cash flows of a bond. On an MBS, for instance, stating, “The bond has an expected average life of 5 years” is very common and very informative, though not in assessing risk. It gives the bondholder an idea of what to expect on principal cash flows if market interest rates remain stable. A common mistake occurs when the bond manager states, “The bond has a duration of 4.” The bond manager typically intends to refer to risk when using the generic term “duration.” Very often, the “duration” he/she is referring to is *modified duration* associated with the *expected* average life. This is virtually worthless from an interest rate risk standpoint. Interest rate risk should refer to what happens to market value when interest rates move adversely (higher). Risk within a 1% band (+/- 1% market interest rate shifts), for instance, is the market value change if rates *rise* 1%. Never mind what market value is, under expected or lower than expected interest rate scenarios, since neither represents risk.

In the above example, say the expected average life increases to 8 years if market interest rates rise 1%. A corresponding *modified duration* to this average life may be 5.75, which would be a closer approximation of risk, but still with

limitations. A key aspect of callable and pre-payable bonds is they carry additional yield versus non-callable and non-pre-payable bonds due to options granted to the bond issuer (or homeowners on MBS). As market prices decrease due to higher market interest rates, these options lose value, thus less additional yield versus non-callable and non-pre-payable bonds is required. This is “spread tightening.” This translates into less of a market value decrease than *modified duration* would suggest. A logical estimate to market value loss at 1% higher interest rates would be 5.25% in this example. If the average life decreases to 2 years with 1% lower market interest rates and market value is determined to increase 1.5%, the *effective duration* would be 3.375 (the average of 5.25 and 1.5). *Effective duration* of 3.375 and *modified duration* of 4, both commonly and mistakenly referred to in risk terms as “duration,” will lead the bond manager down the wrong path of assessing what he/she thinks the bond’s risk is. The bond’s risk is 5.25%. This means the bond should lose 5.25% of its current market value if interest rates immediately rise 1%.

### ***Summary of common misuses of “duration”***

Bond managers commonly make mistakes when using the widely and loosely used term “duration” when referring to interest rate risk. Two of the most common are:

- 1) The use of *modified duration* associated with the *expected* average life (or maturity or call date, whichever is currently expected). Expected interest

rates are not the “risk” scenario. *Higher* than expected interest rates cause interest rate risk, not expected rates.

- 2) The use of *effective duration*, which is too low because it averages in the reward portion (from lower interest rates), which is the *opposite* of risk. The average of market value changes, occurring with higher than expected interest rates and lower than expected interest rates, does not measure risk. *Higher* than expected interest rates alone cause interest rate risk, not the average of both higher and lower rates.

Every bond manager must be keenly aware of the true interest rate risk of a bond, not just what is expected, or averaged with beneficial scenarios.

### ***Summary to background***

*Modified duration* and *convexity* on non-callable and non-pre-payable bonds can be calculated without already knowing the percentage price changes, while *effective duration* and *effective convexity* on callable and pre-payable bonds cannot be calculated without already knowing the percentage price changes. This is a huge difference between the two pairs of interest rate risk measurements. Unfortunately, the more complicated pair of *effective duration* and *effective convexity* has been the only interest rate risk and reward measurements relevant for today's prevalent callable and pre-payable bonds. *Modified duration* and *convexity* are really only useful on non-callable and non-pre-payable bonds.

*Effective duration* and *effective convexity* are simply ways of presenting interest rate risk and reward in a manner that is consistent with its non-callable-equivalent measurements of *modified duration* and *convexity*, even though the two pairs have drastically different origins. If it were not for the useful measurements of Macaulay's duration and modified duration on non-callable bonds, *effective duration* on callable and pre-payable bonds would have probably never been used. The only logical reason is the continued use (and general application of) the term "duration." The information that *effective duration* provides on callable and pre-payable bonds is simply too indirect of a percentage price change gauge to manage by, thus *effective convexity* is needed to complete the full story. However, the process of merging and interpreting *effective duration* and *effective convexity* to reach the ultimate goal of predicting interest rate risk and reward (in the form of percentage price decrease and increase) is more complicated than it needs to be. There must be an easier way.

***MAIN SUBJECT MATTER:***

***Risk Duration, Reward Duration, and Duration Ratio***

Alas, I am introducing a much more intuitive, easy to understand, less complicated and useful way of representing interest rate risk and reward for callable and pre-payable bonds. Risk Duration and Reward Duration will greatly clarify and simplify the way interest rate risk and reward measurements are presented. The fact that a fixed income portfolio manager must have a

market pricing source (simulator) for callable and pre-payable bonds up front has not changed, but instead of having *effective duration* and *effective convexity* represent percentage market price changes in a very roundabout way, one can now use ***Risk Duration*** and ***Reward Duration*** to represent percentage market price changes in a most direct way.

***Risk Duration*** is defined as the up interest rate scenario's percentage price change. It is as simple as that! For consistency sake, the assumed negative sign is removed, and is then multiplied by 100 to have the same convention as other “duration” measurements. ***Risk Duration*** focuses only on the negative percentage price movement that is associated with higher interest rates. The term “risk” is used because higher interest rates represent potential market value loss.

***Reward Duration*** is defined as the down interest rate scenario's percentage price change. It is as simple as that! For consistency sake, the percentage price change is multiplied by 100 to have the same convention as other “duration” measurements. ***Reward Duration*** focuses only on the positive percentage price movement that is associated with lower interest rates. The term “reward” is used because lower rates represent potential market value gain.

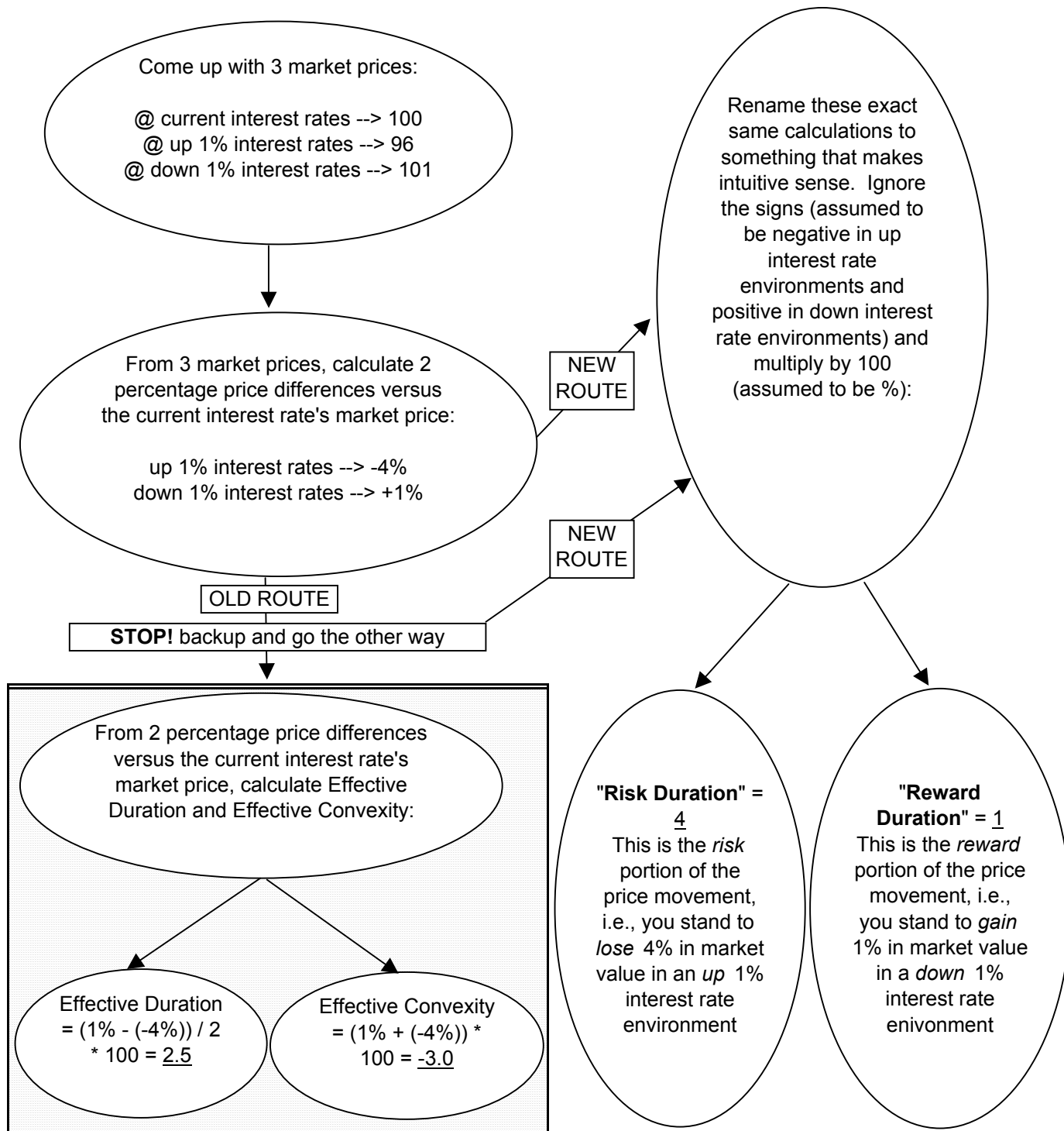
Note: It is possible to have a negative sign attached to ***Risk Duration*** or ***Reward Duration***. This would imply the opposite of the expected percentage

price movement, i.e., negative **Risk Duration** would mean increase in value in higher interest rate scenario and negative **Reward Duration** would mean decrease in value in lower interest rate scenario.

Adding a final chapter to the “two birds with one stone” analogy, on callable and pre-payable bonds, two stones will always be needed to kill the two birds. Instead of aiming directly overhead (12:00 or high noon, i.e., **effective duration**) and making two fairly cumbersome aim adjustments with the help of **effective convexity** to 11:00 and 1:00, imagine being told to simply throw one stone to 11:00 (**Risk Duration**) and throw the other stone to 1:00 (**Reward Duration**). Now that is hard to inadvertently mismanage.

<<The next page contains a flow chart illustrating content from above info>>

This is how you come up with and use Risk Duration and Reward Duration:



A third new measurement, **Duration Ratio**, is simply **Reward Duration / Risk Duration**. It is not needed in determining percentage price changes if **Risk Duration** and **Reward Duration** are provided. However, it is a nice, convenient measure of a bond's ability to increase in price versus its price risk. **Duration Ratio** (also known as "bond quality ratio," "quality ratio," and "Allen's convexity") is closely related to the much more elusive and complicated concept of **effective convexity** (a.k.a. "convexity"). **Duration Ratio** is a natural replacement for **effective convexity**. While **Duration Ratio** can be used on non-callable bonds, such as Treasury notes, it is much more useful on callable bonds, such as bonds with call features, mortgage or other asset backed securities with principal prepayment options, or any other bond where principal cash flow can vary ("callable bonds").

**Duration Ratio** is a measurement of quality (hence "quality ratio" and "bond quality ratio"). The higher the percentage, the better the bond quality. "Quality" in this context is a bond's ability to increase in value relative to its potential to lose value given equal and opposite shifts in market interest rates. It is not a credit quality measurement. If the term "quality ratio" were not already used (it is a sales ratio where  $\text{quality ratio} = 1 - \text{sales returns and allowances} / \text{sales}$ , according to Google), this would be a very appropriate term. As such, **Duration Ratio** is the official name instead of *quality ratio*. "Allen's convexity" is synonymous.

Example of **Duration Ratio**:

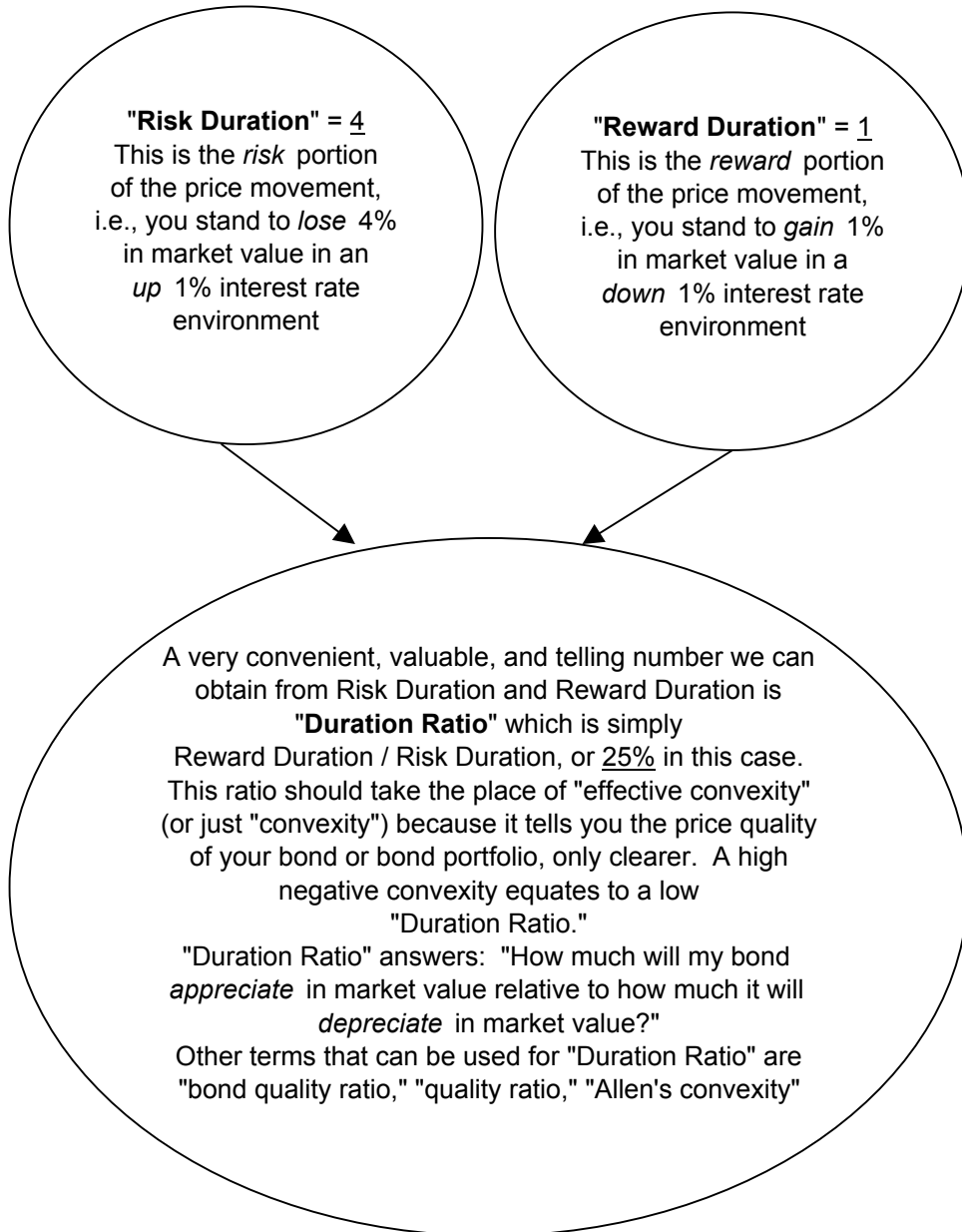
Consider a callable bond with 3-year maturity and 1-year call option that is currently worth 100. Rigorous analyses such as option adjusted calculations using Monte Carlo simulations or other proprietary simulation models used to estimate the bond's market value at various interest rate scenarios must be used. If the bond manager, or another source, determines that the market price would be 98 if interest rates rose 1%, then the percentage price change would be -2%. If it is determined that the market price in an equal and opposite move in interest rates (down 1%) results in a market price of 101, the percentage price change would be +1%. Now to **Duration Ratio**:  $+1\% / -(-2\%) = 50\%$ , or simply "1" **Reward Duration** / "2" **Risk Duration** = 50%. This means the bond's market price stands to gain 50% of the value it stands to lose given equal and opposite rate moves (1% in this case).

Assume **Risk Duration** = "2" in the given rate assumption range (+/-1% in this case). If the bond stands to increase in value 0% of the amount it stands to lose ( $0\% * 2\% = +0\%$ , or "0" **Reward Duration**), it is known that this bond has poor quality dynamics (**Duration Ratio** = 0%), thus would require a higher yield to compensate for the weak risk/reward tradeoff (or reward/risk in the case of **Duration Ratio**). If the bond stands to increase in value 100% of the amount it stands to lose ( $100\% * 2\% = +2\%$ , or "2" **Reward Duration**), it is known that this bond has great quality dynamics (non-callable and/or very deep discount) (**Duration Ratio** = 100%), thus would require a lower yield as a result of the

strong risk/reward tradeoff (or reward/risk). If the bond stands to increase in value 50% of the amount it stands to lose ( $50\% * 2\% = +1\%$ , or “1” **Reward Duration**), it is known that this is a mid-level quality bond (**Duration Ratio** = 50%), thus would expect a yield somewhere in between the yields of **Duration Ratio** = 0% and **Duration Ratio** = 100% bonds.

<<The next page contains a flow chart illustrating content from above info>>

This is how you can tie Risk Duration and Reward Duration together to create a meaningful ratio that is much more intuitive, logical, and easier to understand than "convexity"



It is important to know that determining market prices on callable bonds in various interest rate scenarios is not obtainable in a static mathematical formula. There are assumptions such as spread tightening and widening, implied volatility, prepayment speeds, etc. that help determine market and would-be market prices under various interest rate scenarios. **Effective duration** and **effective convexity** on callable bonds require market prices in current, up and down interest rate scenarios to be determined before calculation. **Risk Duration**, **Reward Duration**, and **Duration Ratio** also require market prices in current, up and down interest rate scenarios to be determined before calculations.

To add precision, **Risk Duration**, **Reward Duration**, and **Duration Ratio** should contain a designated interest rate scenario (1%, 2%, 3%, etc.). A weakness in the currently used “duration” and “convexity” numbers (duration, **modified duration**, Macaulay's duration, adjusted duration, risk, **effective duration**, convexity, **effective convexity**) is they all assume 1% interest rate movements around current interest rates. **Risk Duration1**, **Reward Duration1**, **Duration Ratio1**, **Risk Duration2**, **Reward Duration2**, **Duration Ratio2**, **Risk Duration3**, **Reward Duration3**, **Duration Ratio3**, etc., tell the full, more accurate, story. Any number may be added to **Risk Duration**, **Reward Duration**, and **Duration Ratio**, such as .25, .5, 4, 5, etc. Importantly, adjustments are not made to base everything off of 1 like “duration” and “convexity” numbers do. For instance, a 3% interest rate scenario **effective duration** measurement of 4 actually suggests a loss of 12% ( $4 * 3 = 12$ ) before the “convexity” adjustment. The simpler **Risk**

**Duration3** of -12%, or “12” stands to lose just that, 12%. If the interest rate scenario designation is omitted, it should be assumed to be 1%.

While one could extrapolate a common “duration” and “convexity” measurement from its assumed 1% base measurement to estimate values at 2% and 3%, on most callable bonds, a large amount of accuracy will be lost. To insure accuracy, each interest rate scenario needs its own, independent calculations for market prices, which determines percentage price changes, which in turn translates into **Risk Duration**, **Reward Duration**, and **Duration Ratio** in the given interest rate scenario.

**Risk Duration**, **Reward Duration**, and **Duration Ratio** do not contain entirely new calculations, but are much more intuitive, understandable, and usable ways of presenting risk (and reward) assessment measurements versus **effective duration** and **effective convexity**. **Duration Ratio** cannot be calculated directly from **effective convexity**, nor can **Risk Duration** or **Reward Duration** be calculated from **effective duration**, but **Risk Duration**, **Reward Duration**, and **Duration Ratio** can be calculated from the pair of **effective duration** and **effective convexity**.

The pair of **Risk Duration** and **Reward Duration** or the pair of **Risk Duration** and **Duration Ratio** can be used as a logical replacement for the pair of **effective duration** and **effective convexity**. I prefer the pair of **Risk Duration** and **Duration**

**Ratio.** One directly shows actual risk, while the other directly provides the price movement quality. Either way, it is extremely easy to calculate the third figure, given any two of **Risk Duration**, **Reward Duration**, and **Duration Ratio**, so it does not really matter which pair (or all three) is used.

If using the pair of **effective duration** and **effective convexity**, bond managers really have to know what they are doing to unscramble each definition and must know both **effective duration** and **effective convexity** in order to surmise what their risk, reward, and quality is. Given a specific interest rate range (+/- 1%, for example), what a bond manager really wants to know about a bond, other than its yield, is its potential risk and its potential reward, which together make up the bond's potential price movement quality.

A bond manager can obtain the bond's potential risk by either: (A): **Risk Duration** OR (B): [ $.5 * \text{effective convexity}$ ] - **effective duration**].

A bond manager can obtain the bond's potential reward by either: (A): **Reward Duration** OR (B): [**effective duration** +  $\{.5 * \text{effective convexity}\}$ ].

A bond manager can obtain the bond's potential price movement quality by either: (A): **Reward Duration** / **Risk Duration** OR (B): [**effective duration** +  $\{.5 * \text{effective convexity}\}$ ] / [ $\{.5 * \text{effective convexity}\}$  - **effective duration**].

A bond manager can make the choice of which method, (A) or (B), is most understandable and most useful, therefore most beneficial in managing interest rate risk.

### ***Applications of measurements by themselves***

**Risk Duration** can be used by itself to get the full picture of the bond's interest rate risk, though it is useless in gauging a bond's interest rate reward. On the other hand, **effective duration** does not do an acceptable job of assessing a bond's interest rate risk (or reward). For example, is **effective duration** of 2 “risky” or not? If it is determined that anything less than a 3% loss in market value is acceptable given a 1% rise in market interest rates, the answer to the previous question is “no,” it is not “risky.” But what if the other half of the equation, **effective convexity**, is -4? The bond stands to lose 4% ( $\{.5 * -4\} - 2 = -4$ ) in market value on a 1% rise in interest rates, which is considered “risky.” The **effective duration** of 2, in this case paired with -4 convexity, therefore, is too risky where initially, armed with only **effective duration** of 2, it was thought to be just fine.

What if the other half of the equation, **effective convexity**, is 0? The bond stands to lose 2% ( $\{.5 * 0\} - 2 = -2$ ) in market value on a 1% rise in interest rates, which is not considered “risky.” The **effective duration** of 2, in this case paired with 0 convexity, therefore, is not too risky. **Effective duration** by itself is not adequate in assessing risk on callable or pre-payable bonds. **Risk Duration** by itself, on

the other hand, tells the entire story of the bond's interest rate risk. In this example, any **Risk Duration** greater than 3 is unacceptable, while all other readings are acceptable. That is all that is needed using the **Risk Duration** method. **Risk Duration** is not dependent on anything else, like convexity (or **Duration Ratio**), to make an informed decision.

**Duration Ratio** can be used by itself to get a good idea of the bond's performance quality, while **effective convexity** does not do a good job of this. For instance, is **effective convexity** of -2 good or bad? If the bond's **effective duration** is 10, -2 **effective convexity** is pretty good (82% **Duration Ratio** and -11% **Risk Duration**, or "11"), meaning percentage price changes from current market price are -11% and +9% ( $9 / 11 = 82\%$ ). If the bond's **effective duration** is 1, -2 **effective convexity** is not good (0% **Duration Ratio** and -2% **Risk Duration**, or "2"), meaning percentage price changes from current market price are -2% and +0% ( $0 / 2 = 0\%$ ). Thus, **effective duration** needs to be known before gauging whether **effective convexity** is bad or relatively harmless. If using just **Duration Ratio**, the bond's performance quality is known without depending on **Risk Duration** or **effective duration**. In the above examples, the **Duration Ratio** of 82% has a high performance quality, while the 0% measurement has a low performance quality.

Individually, **effective duration** and **effective convexity** are not very valuable. Fixed income portfolio managers cannot possibly make prudent decisions armed

only with one of these measurements. **Risk Duration** and **Duration Ratio**, on the other hand, are very valuable individually. Bond managers commonly set maximum losses (5% market value loss at +1% market interest rates, for instance), which can be set in policy with “**Risk Duration** must be less than or equal to 5.” Bond managers can now also set an independent policy limit or target using **Duration Ratio**. “A minimum **Duration Ratio** on the bond portfolio of 40%”, for instance, could be a valuable policy limit or target. A maximum **effective duration** of 3.5, for example, is too nebulous to be considered a prudent policy limit or target. If just using **effective duration** (or **modified duration**), a fixed income portfolio manager is assured to mismanage, provided the portfolio consists of an appreciable amount of callable or pre-payable bonds. A minimum **effective convexity** of negative 3, for example, is also too nebulous to be considered a prudent policy limit or target and is likely to result in mismanagement.

### ***Portfolio management applications***

The aforementioned policy example for **effective duration** (maximum of 3.5) and **effective convexity** (minimum of -3) put together still does not accomplish what the roughly equivalent policy example of **Risk Duration** (maximum of 5) and **Duration Ratio** (minimum of 40%) does. Both limit risk to 5%, but what if we had **effective duration** = 1 and **effective convexity** = -2? Both are within the limits, but the quality, or **Duration Ratio**, would be a dismal 0%. This would not pass our **Duration Ratio** minimum of 40%. There is no way to arrange a policy that

unilaterally limits both **effective duration** and **effective convexity** that will accomplish what the no-nonsense, worthwhile and beneficial sample policy of a 5% maximum loss (***Risk Duration1***) and a 40% or better price appreciation versus price risk (***Duration Ratio1***) will. Another broader policy sample is ***Risk Duration3*** < 15 and ***Duration Ratio3*** > 40%. Other than depository institutions (banks, etc.) addressing liquidity requirements, ***Risk Duration*** and ***Duration Ratio*** are the two most powerful risk management statistics available to fixed income portfolio managers.

This paper, created solely by Mike Allen of Birmingham, AL, USA, is registered at <http://www.ssrn.com>